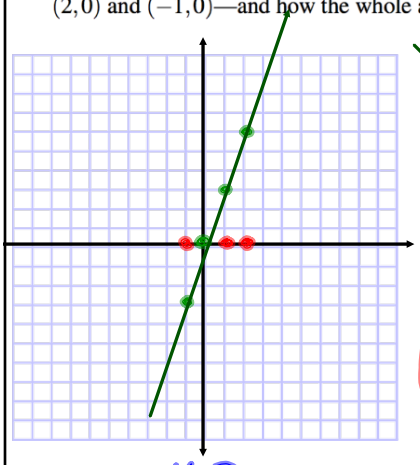
5. The matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ yields a ***shearing*** transformation, which leaves the y-axis unchanged. Sketch its effect on the x-axis, by indicating what happens to $(1,0)$ and $(2,0)$ and $(-1,0)$—and how the whole axis is transformed.



$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

$x=0$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 3x+y \end{bmatrix}$$

$$(0,y) \longrightarrow (0,y)$$

36. (a) What matrix transforms $(1,0)$ into $(2,5)$ and transforms $(0,1)$ to $(1,3)$?
(b) What matrix transforms $(2,5)$ to $(1,0)$ and $(1,3)$ to $(0,1)$?
(c) Why does no matrix transform $(2,6)$ to $(1,0)$ and $(1,3)$ to $(0,1)$?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \qquad \begin{bmatrix} 2 & b \\ 5 & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$a=2$, $c=5$ $\qquad$ $b=1$, $d=3$

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$2a+5b=1$ $\qquad$ $2c+5d=0$

$a+3b=0$ $\qquad$ $c+3d=1$

$b=-1$ $\qquad$ $d=2$

$a=3$ $\qquad$ $c=-5$

$$\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

38 b) What matrix transforms (2,5) to (1,1) and (1,3) to (0,2)?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$2a+5b=1$

$2c+5d=1$

$a+3b=0 \Rightarrow a=-3b$

$c+3d=2$

$c=2-3d$

$2(-3b)+5b=1$

$-b=1$

$b=-1$

$a=3$

$2(2-3d)+5d=1$

$4d=1$

$d=3$

$c=-7$

$$\begin{bmatrix} 3 & -1 \\ -7 & 3 \end{bmatrix}$$

38 b) What matrix transforms (2,5) to (1,1) and (1,3) to (0,2)?

A^{-1} → B: (1,0) → (0,1)

$$A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \qquad A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$BA^{-1}$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ -7 & 3 \end{bmatrix}$$

$$R\cdot\begin{bmatrix}2\\6\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$R\cdot\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$$

$$R(\vec{a})+R(\vec{b})=R(\vec{a}+\vec{b})$$